EXERCISE – V HINTS & SOLUTIONS

Sol.1 C

Coeff. of x = 3 $\Rightarrow {}^mC_1 1 .1 + {}^nC_1 (-1) . 1 = 3$

$\Rightarrow m - n = 3$ .... (i)

Coeff. of $x^2 = -6 \Rightarrow 1. {}^mC_2\, 1 + 1. {}^nC_2 (-1)^2 + {}^mC_1\, 1.1. {}^nC_1 (-1).1 = -6$

$$\Rightarrow \frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn = -6$$

$\Rightarrow m^2 - m + n^2 - n - 2mn + 12 = 0$

$\Rightarrow (m - n)^2 - (m + n) + 12 = 0$

$\Rightarrow 3^2 - (m + n) + 12 = 0$ From (i)

$\Rightarrow m + n = 21$ ... (ii)

From (i) & (ii) $\Rightarrow 2m = 24 \Rightarrow m = 12$

Sol.2 D

$$\text{Given } \binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2}$$

$$= \binom{n}{r} + \binom{n}{r-1} + \binom{n}{r-1} + \binom{n}{r-2}$$

$$= \binom{n+1}{r} + \binom{n+1}{r-1} = \binom{n+2}{r}$$

Sol.3 LHS $= \binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \ldots + \binom{m}{m}$

$$\binom{m}{m} + \binom{m+1}{m} + \ldots + \binom{n-1}{m} + \binom{n}{m}$$

$$\left\{\because \binom{m}{m} = \binom{m+1}{m+1}\right\}$$

$$= \binom{m+1}{m+1} + \binom{m+1}{m} + \ldots + \binom{n-1}{m} + \binom{n}{m}$$

$$= \binom{m+2}{m+1} + \binom{m+2}{m} + \ldots + \binom{n-1}{m} + \binom{n}{m}$$

$$= \binom{n-1}{m+1} + \binom{n-1}{m} + \binom{n}{m}$$

$$= \binom{n}{m+1} + \binom{n}{m} = \binom{n+1}{m+1} = \text{R.H.S.}$$

Hence L.H.S. $= {}^nC_m + 2\, {}^{n-1}C_m + 3\, {}^{n-2}C_m + \ldots. + (n - m + 1)\, {}^mC_m$

$$\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \ldots + \binom{m}{m}$$

$$+ \left[\binom{n-1}{m} + \binom{n-2}{m} + \ldots + \binom{m}{m}\right]$$

$$+ \left[\binom{n-2}{m} + \ldots. + \binom{m}{m}\right]$$

$$+ \binom{m}{m}$$

$$= \binom{n+1}{m+1} + \binom{n}{m+1} + \binom{n-1}{m+1} + \binom{m}{m}$$

$$= \binom{m}{m} + \ldots + \binom{n-1}{m+1} + \binom{n}{m+1} + \binom{n+1}{m+1} = \binom{n+2}{m+2}$$

= R.H.S.

Sol.4 Largest coeff. of $(1 + x)^n$

If ${}^nC_0 + {}^nC_1 + \ldots + {}^nC_n = 4096$

$\Rightarrow 2^n = 2^{12} \Rightarrow n = 12$

Largest coeff. ${}^{12}C_r$ if $r = \frac{12}{6} \Rightarrow r = 6 \therefore {}^{12}C_6$

Sol.5 B

Given that $T_5 + T_6 = 0$

${}^nC_4\, a^{n-4} (-b)^4 + {}^nC_5\, a^{n-5} (-b)^5 = 0$

$\Rightarrow a^{n-5}b^4 [{}^nC_4\, a - {}^nC_5 b] = 0$

$\Rightarrow {}^nC_4 a = {}^nC_5 b$ $(\because a \neq 0, b \neq 0)$

$$\Rightarrow \frac{a}{b} = \frac{{}^nC_5}{{}^nC_4} = \frac{n-4}{5}$$

Sol.6

We know (x – a) (x – b) (x – c)

$= x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc$

Max degree of polynomial is 50, so coeff. of x^{49}

$$= -\left(\frac{c_1}{c_0} + 2^2\frac{c_2}{c_1} + 3^2\frac{c_3}{c_2} + \ldots.. + 50^2\frac{c_{50}}{c_{49}}\right)$$

$$= -\sum_{r=1}^{50} r^2 \left(\frac{{}^{50}c_r}{{}^{50}c_{r-1}}\right) = -\sum_{r=1}^{50} r^2 \frac{(50-r+1)}{r}$$

$$= -\sum_1^{50} 51(r) + \sum_{r=1}^{50} r^2 = \sum_1^{50} r^2 - 51\sum_1^{50} r$$

$$= \frac{50(50+1)(100+1)}{6} - 51 . \frac{50.51}{2}$$

$$= \frac{50.51}{6}[101 - 51.3] = -\frac{50.51}{6}.52 = -25.17.52$$

$= -22100$

Sol.7 C

$$\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$$

$$=\binom{10}{0}\binom{20}{m}+\binom{10}{1}\binom{20}{m-1}+....+\binom{10}{m}\binom{20}{0}$$

= Total no. of ways of m thing selection out of 30 where 10 are alike & other 20 are alike different to previous 10 = $^{30}C_m$ is max. if 30 = 2m $\Rightarrow$ m=15

Sol.8(a) A

coeff. of t^{24} in $(1 + t^{12})(1 + t^{24})(1 + t^2)^{12}$

= coeff. of t^{24} in $(1 + t^{12} + t^{24} + t^{36})(1 + t^2)^{12}$

$= 1\ .\ ^{12}C_0 + 1.^{12}C_{12} + 1.^{12}C_6$

$= 1 + 1 + {}^{12}C_6 = {}^{12}C_6 + 2$

Sol.8(b) L.H.S. $= T_{r+1} = (-1)^r \binom{n}{r}\binom{n-r}{k-r}.2^{k-r}$

$$=\frac{n!}{r!\,(n-r)!}\cdot\frac{(n-r)!}{(k-r)!\,(n-k)!}\ (-1)^r\ 2^{k-r}$$

$$=\frac{n!}{k!\,(n-k)!}\cdot\frac{k!}{r!(k-r)!}\ (-1)^r\ 2^{k-r}$$

$$=\sum_{r=0}^{k}\ {}^nC_k\ {}^kC_r\ (-1)^r\ 2^{k-r}$$

$$={}^nC_k\ \sum_{r=0}^{k}\ {}^kC_r\ (-1)^r\ 2^{k-r}$$

$$={}^nC_k\ \ (-1+2)^k = {}^nC_k\ .\ 1^k = \binom{n}{k} = \text{R.H.S.}$$

Sol.9 D

$^{n-1}C_r = (k^2 - 3).^nC_{r+1}$

$\Rightarrow\ ^{n-1}C_r = (k^2 - 3)\left(\frac{n}{r+1}\right)\ ^{n-1}C_r$

$\Rightarrow\ 1 = (k^2 - 3)\left(\frac{n}{r+1}\right) \Rightarrow \left(\frac{r+1}{n}\right) = k^2 - 3$

we know $n \ge (r + 1) \Rightarrow \left(\frac{n}{r+1}\right) \ge 1$

$\Rightarrow \left(\frac{r+1}{n}\right) \le 1 \Rightarrow k^2 - 3 \le 1 \Rightarrow k^2 \le 4$

$\Rightarrow k \in [-2, 2]$ ....(i)

But $^{n-1}C_r = (k^2 - 3)\ ^nC_{r+1}$

$\because\ ^{n-1}C_r > 0\ \ \&\ \ ^nC_{r+1} > 0 \qquad \therefore\ k^2 - 3 > 0$

$\Rightarrow k \in (-\infty, -\sqrt{3}) \cup (\sqrt{3} \cup \infty)$ .....(ii)

from (i) & (ii) $\Rightarrow k \in [-2, -\sqrt{3}) \cup (\sqrt{3}, 2]$

Sol.10 A

Let $(1 + x)^{30} = {}^{30}C_0 + {}^{30}C_1\, x + {}^{30}C_2\, x^2 + ...+ {}^{30}C_{20}\, x^{20} ...... + {}^{30}C_{30}\, x^{30}$

$(x - 1)^{30} = {}^{30}C_0\, x^{30} - {}^{30}C_1\, x^{29} + {}^{30}C_2\, x^{28} - .... + {}^{30}C_{20}\, x^{20} ...... + {}^{30}C_{30}$

multiply both equation and compair coeff. of x^{20}

$$\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+$$

$$\binom{30}{2}\binom{30}{12}-....+\binom{30}{20}\binom{30}{30}$$

= coeff of x^{20} in $(x^2 - 1)^{30}$

$= {}^{30}C_r\, (x^2)^{30-r}\, (-1)^r = {}^{30}C_r\ x^{60-2r}(-1)^r$

$= {}^{30}C_{20}\, (-1)^{20} = {}^{30}C_{20} = {}^{30}C_{10}$

$\{\because 60 - 2r = 20 \Rightarrow r = 20\}$

Sol.11 C

Case–I 1111123 $\Rightarrow \frac{7!}{5!} = 7 \times 6 = 42$

Case–II 1111222 $\Rightarrow \frac{7!}{4!\,3!} = 7 \times 5 = 35$

Total = 42 + 35 = 77

Sol.12 D

$$\sum_{r=1}^{10} A_r\,(B_{10}\,B_r - C_{10}A_r)$$

$$=\sum_{r=1}^{10}\ {}^{10}C_r\left({}^{20}C_{10}{}^{20}C_r - {}^{30}C_{10}\,.\,{}^{10}C_r\right)$$

$$\left\{\because C_0^2 + C_1^2 + C_2^2 + ... + C_n^2 = {}^{2n}C_n\right\}$$

$$= {}^{20}C_{10}\sum_{r=1}^{10}\ {}^{10}C_r\ {}^{20}C_r - {}^{30}C_{10}\sum_{r=1}^{10}\left({}^{10}C_r\right)^2$$

$= -\,{}^{20}C_{10} + {}^{30}C_{10} = C_{10} - B_{10}$

$\because\ (1+ x)^{10} = {}^{10}C_0 + {}^{10}C_1\, x + {}^{10}C_2\, x^2 +...+ {}^{10}C_9\, x^9 + {}^{10}C_{10}\, x^{10}$

$(1 + x)^{20} = {}^{20}C_0\, x^{20} + {}^{20}C_1\, x^{19} + {}^{20}C_2\, x^{18} +........ + {}^{20}C_9\, x^{11} + {}^{20}C_{10}\, x^{10} - {}^{20}C_{20}\, x^2 + {}^{20}C_{19}x + {}^{20}C_{18}\, x^0 +......+ {}^{20}C_{11}\, x^9$

Multiply & get coeff. of x^{20} in $(1 + x)^{30}$

$= {}^{10}C_0\, {}^{20}C_0 + {}^{10}C_1\, {}^{20}C_1 + {}^{10}C_2\, {}^{20}C_2 + ...+ {}^{10}C_{10}\, {}^{20}C_{10}$

$= {}^{30}C_{20} = {}^{30}C_{10}$